Name Sara Zamzam, , ID 88202

Quiz one, MTH 221 , Fall 2022

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. Find the least positive integer x such that

$$x \ (mod\ \underset{m_1}{7}) = \underset{r_1}{6}$$
$$x \ (mod\ \underset{m_2}{11}) = \underset{r_2}{9}$$
$$x \ (mod\ \underset{m_3}{5}) = \underset{r_3}{3}$$

$gcd(7,11)=1; gcd(7,5)=1, gcd(11,5)=1$

We can use CRT

$m = m_1 m_2 m_3 = (7)(11)(5) = 385$

$b_1 = \frac{m}{m_1} = \frac{\cancel{7}\times 11\times 5}{\cancel{7}} = 55$

$b_2 = \frac{m}{m_2} = \frac{7\times \cancel{11}\times 5}{\cancel{11}} = 35$

$b_3 = \frac{m}{m_3} = \frac{7\times 11\times \cancel{5}}{\cancel{5}} = 77$

Solve $b_1 d_1 = 1$ over Z_{m_1}

$55 d_1 = 1$ over Z_7

$6 d_1 = 1$ over Z_7

$d_1 = 6$

$b_2 d_2 = 1$ over Z_{m_2}

$35 d_2 = 1$ over Z_{11}

$2 d_2 = 1$ over Z_{11}

$d_2 = 6$

$b_3 d_3$ over Z_{m_3}

$77 d_3 = 1$ over Z_5

$2 d_3 = 1$ over Z_5

$d_3 = 3$

Least positive x; $0 \leq x < 385$

$x = [b_1 d_1 r_1 + b_2 d_2 r_2 + b_3 d_3 r_3] (mod\ m)$

$x = [(55)(6)(6) + (35)(6)(9) + (77)(3)(3)] (mod\ 385)$

$x = 4563\ (mod\ 385)$

$x = 328$

$\frac{10}{10}$

QUESTION 2. Solve over Z_9 $6x = 3$ over Z_9.

$a=6, b=3, n=9$

0.5 $d = gcd(a,n) = gcd(6,9) = 3$

Is $3|3$? Yes, there are 3 distinct solutions

$f = \frac{9}{3} = 3$

1.5 $x_1 = 2$

1.5 $x_2 = 2+3 = 5$

1.5 $x_3 = 2+2(3) = 8$

Solution set $= \{2,5,8\}$ ✓ $\frac{5}{5}$

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Name Ahmad Alsaleh, ID 93749

Quiz Two MTH–213 , Fall 2022

Ayman Badawi

Score = $\boxed{\frac{15}{15}}$

$2 \cdot 13$
$3 \cdot 5 \cdot 13$

QUESTION 1. Let $n = 26$ and $m = 195$
Find $gcd(n, m) = gcd(26, 195)$

4/4

$$26 \overline{)\,195\,}^{\,7} \qquad 195 - 182 = 13$$

$$\Rightarrow \overset{\text{gcd}}{\boxed{13}} \overline{)\,26\,}^{\,2} \qquad 26 - 26 = 0 \rightarrow \text{stop!!}$$

$\boxed{gcd(n, m) = 13}$ ✓ ✓

Find LCM[26, 195]

3/3

$$lcm = \frac{26 \cdot 195}{gcd(26, 195)} = \boxed{390}$$ ✓ ✓

Find $(11^{6\phi(25)+2})$ $(mod\ \ 25)$

4/4

$11^{\phi(25)} (mod\, 25) = 1$ since $gcd(11, 25) = 1$

$11^{6\phi(25)+2} (mod\ 25) = 11^{6\phi(25)} \cdot 11^2 (mod\ 25)$

$= [\underbrace{11^{\phi(25)}}_{1}]^6 \cdot 11^2 \ (mod 25) = 11^2 (mod\ 25) = 121 (mod 25) = \boxed{21}$ ✓ ✓

Let $n = 104$ and $k = 4$ and $D = \{m \in Z_{104} \mid gcd(n, m) = 4\}$
Find $|D|$.

4/4

$|D| = \phi(\frac{104}{4}) = \phi(26) = \phi(2 \times 13) = (2-1)(13-1) = 1 \times 12 = \boxed{12}$ ✓ ✓

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Name Sara Zamzam, ID 88202

Quiz Three MTH–213 , Fall 2022

Ayman Badawi

Score = $\frac{\quad}{15}$

15/15

QUESTION 1. (16 points)

(i) Let x be an even integer and y be an odd integer. Prove that $x + y$ is an odd integer.

$x = 2m, m \in \mathbb{Z}$

$y = 2n+1, n \in \mathbb{Z}$

$x + y = 2m + 2n + 1 = 2\underbrace{(m+n)}_{w \in \mathbb{Z}} + 1 = 2w + 1$

$2w+1$ is an odd integer $\in \mathbb{Z}$. ✓ 5/5

(ii) Use the 4th method and prove that $\sqrt{27}$ is an irrational number.

Deny. $\sqrt{27}$ is rational.

Hence $(\sqrt{27} = \frac{a}{b})$'s.t $\gcd(a,b) = 1$ and $b \neq 0$

$27 = \frac{a^2}{b^2}$ $\gcd(a^2, b^2) = 1$

odd $= \frac{a^2}{b^2}$ a^2 and b^2 are odd

$a = 2m+1$; $m \in \mathbb{Z}$

$b = 2n+1$, $n \in \mathbb{Z}$

$27 = \frac{(2m+1)^2}{(2n+1)^2} = \frac{4m^2+4m+1}{4n^2+4n+1}$

$(27 \cdot 4n^2 + 27 \cdot 4n + 26 = 4m^2 + 4m)/4$

$\underbrace{\left(\underbrace{27n^2 + 27n}_{\in \mathbb{Z}} + \frac{26}{4}\right)}_{\notin \mathbb{Z}} = \underbrace{m^2 + m}_{\in \mathbb{Z}}$ a contradiction. ✓ ✓

Hence, our denial is invalid. $\sqrt{27}$ is irrational. ✓

5/5

(iii) Prove that $(1 + \sqrt{27})^2$ is an irrational number. (use (ii))

$(1 + \sqrt{27})^2 = 1 + 2\sqrt{27} + 27 = 28 + 2\sqrt{27}$

Through (ii), $\sqrt{27}$ is irrational.

Hence, $2\sqrt{27}$ is irrational,

$(1 + \sqrt{27})^2 = 28 + 2\sqrt{27}$ is also irrational. ✓

5/5

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Name Nabil Bukhary, ID 93375

MTH 213 , Fall 2022, 1–1

Quiz Four MTH–213 , Fall 2022

Ayman Badawi

Score = $\boxed{\frac{15}{15}}$

QUESTION 1. Use Math Induction and prove that $12|(5^{4n}-1)$

$\frac{2}{2}$ 1) We prove for $n=1$. If $n=1$, $5^{4n}-1 = 5^4-1 = 624$

$\frac{624}{12} = 52$, $624 = 12 \times 52$, therefore $12|5^{4n}-1$ is true for $n=1$ ✓

$\frac{2}{2}$ 2) We assume that $12|5^{4n}-1$ is true for some $n=k\geq 1$.
We say $12|5^{4k}-1$ is true. We prove for $n=k+1$ ✓

$\frac{5}{5}$ 3) If $n=k+1$, $5^{4n}-1 = 5^{4(k+1)}-1 = 5^{4k+4}-1$

$\Rightarrow 5^{4k} \cdot 5^4 - 1 = 5^{4k} \cdot 5^4 - 5^4 + 5^4 - 1$

$\rightarrow 5^4(\underbrace{5^{4k}-1}_{\text{Divisible by 12 by step 2}}) + \underbrace{5^4-1}_{\text{Divisible by 12 by step 1}}$ ✓ Therefore $12|5^{4n}-1$ for $n\geq 1$

QUESTION 2. Write T or F

$\frac{6}{6}$

(i) If I am on the moon now, then I am smoking Hooka "arjeela" → T ✓

(ii) $\underbrace{\text{there is an } x \in Q \text{ such that } x^2-2=0}_{F}$ if and only if $\underbrace{2y+5=0 \text{ for some } y \in Z}_{F}$. → T ✓

(iii) If $\underbrace{\text{there is a unique } x \in Z \text{ such that } 2x^2-x=0}_{T}$, then $\underbrace{y^2+1=0 \text{ for some } y \in R}_{F}$. → F ✓

(iv) There is a unique $x \in Q^*$ such that $xy=1$ for every $y \in Z^*$. → F ✓

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Name Hala Maadi , ID 00092771

$\frac{15}{15}$

Quiz 5 MTH–213 , Fall 2022

Ayman Badawi

Score = $\frac{\quad}{15}$

QUESTION 1. Let $A = \{2, 3, 7, \{7\}, \phi\}$ and $B = \{2, \phi, 7\}$

(i) Find $A \cap B$

$A \cap B : \{2, 7, \phi\}$ ✓ 1.5

$A \times B : \{(2,2), (2,\phi), (2,7),$
$(3,2), (3,\phi), (3,7),$
$(7,2), (7,\phi), (7,7),$
$(\{7\},2), (\{7\},\phi), (\{7\},7),$
$(\phi,2), (\phi,\phi), (\phi,7)\}$

$\frac{3}{3}$

(ii) Find $A - B$

$A - B : \{3, \{7\}\}$ ✓ 1.5

elements in A
not in B

(iii) Write down T or F

a. $\{7\} \in A$ T ✓ ~~1.5~~

b. $\{7\} \subset A$ T ✓

c. $\{(3, \phi)\} \in P(A \times B)$. T ✓

d. $\{(7, 2)\} \subset P(A \times B)$. F ✓

e. $\{(3, 7), (\{7\}, 2)\} \subset A \times B$ T ✓

f. $\{7, \phi, 3\} \in P(A)$ T ✓

g. $\{2, 7\} \subset B$ T ✓

h. $\{\phi, 3, 2\} \in P(A)$ T ✓

each = 1.5

$\frac{12}{12}$

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Name Sara Zamzam, ID 88202

MTH 213, Quiz 6

Ayman Badawi

15/15

QUESTION 1. Let $A = \{2, 4, 9, 10, 11, 16, 23, 25\}$ Define "=" on A such that $\forall a, b \in A$, we have $a" = "b$ if $a\ (mod\ 7) = b\ (mod\ 7)$. Then "=" is an equivalence relation (do not show that)

1) Find all equivalence classes of A

$\overline{2} = \{2, 9, 16, 23\}$

$\overline{4} = \{4, 11, 25\}$

$\overline{10} = \{10\}$

✓ 6/6

2) View "=" as a subset of $A \times A$. How many elements does " = " have?

$|"="| = (4 \times 4) + (3 \times 3) + (1 \times 1)$

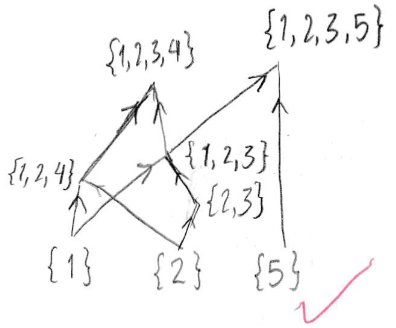
$= 16 + 9 + 1$

$= 26$

✓ 1/1

QUESTION 2. Let $A = \{\{1\}, \{2\}, \{5\}, \{2, 3\}, \{1, 2, 4\}, \{1, 2, 3\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}\}$. Define $\leq$ on A such that $\forall a, b \in A$, we have $a \leq b$ if $a \subseteq b$. Then $\leq$ is a partial order relation on A (do not show that)

1) Draw the Hasse diagram of $\leq$.



✓ 4/4

2) Find

a) $gcd(\{1, 2, 3\}, \{1, 2, 4\})$ = DNE ✓

b) $lcm(\{2\}, \{5\})$ = {1,2,3,5} ✓

c) $lcm(\{1, 2, 3\}, \{1, 2, 4\})$ = {1,2,3,4} ✓

d) $gcd(\{2, 3\}, \{1, 2, 4\})$ = {2} ✓

4/4

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Name Mohamad Jawad Ahmad Kaddoura, ID 00091857

$\frac{15}{15}$

MTH 213, Quiz 7

Ayman Badawi

QUESTION 1.

```
For $k = 3$ to $(n+5)$ do
  $x = k^4 + 3k + 2$
    For $i = 1$ to $(k+1)$ do
    $y = i^2 + 3i + k^2$
    next i $y_1, y_1, y_1, y_1, y_1$
  next k
```

$(3+1+1+1)$

$\frac{7}{7}$ (i) How many arithmetic operations does the code execute?

outer loop: executed $n+5-3+1$ ~~times~~ $= n+3$, no. of arithmetic ops. $= 6(n+3)$

inner loop: ~~ex~~ every time the inner loop runs it is executed $k+1-1+1 = k+1$ times

no. of arithmetic ops. $= 5(k+1)$

first term: for $k=3$ $5(3+1) = 20$ ops.

Last term: for $k = n+5$ $5(n+6)$ ops.

total no. of arithmetic ops. $= 6(n+3) + \left[\frac{20+5(n+6)}{2}\right] \times (n+3)$ ✓

$\frac{1}{1}$ (ii) Find the complexity of the code, i.e., $\mathrm{O}(CODE)$.

$O(\text{code}) = O(n^2)$ ~~$= n^2$~~ ✓

QUESTION 2. Let $a_n = -2a_{n-1} + 15a_{n-2}$, where $a_1 = 1$ and $a_2 = 43$

$\frac{1}{1}$ 1) Find a_3 $a_3 = -2a_2 + 15a_1 = -2(43) + 15(1) = -71$ ✓

2) Find a general formula for a_n.

$\frac{5}{5}$

$a_n + 2a_{n-1} - 15a_{n-2} = 0$

$$\frac{\alpha^n + 2\alpha^{n-1} - 15\alpha^{n-2} = 0}{\alpha^{n-2}} \Rightarrow \alpha^2 + 2\alpha - 15 = 0$$

$(\alpha - 3)(\alpha + 5) = 0$

$\alpha_1 = 3, \alpha_2 = -5$

$a_n = c_1(3)^n + c_2(-5)^n$

$$\left.\begin{array}{l} a_1 = 3c_1 - 5c_2 = 1 \\ a_2 = 9c_1 + 25c_2 = 43 \end{array}\right\} \Rightarrow c_1 = 2, c_2 = 1$$

$\Rightarrow a_n = 2(3)^n + (-5)^n$ ✓

$\frac{1}{1}$ 3) use (2) and find a_3.

$n = 3 \Rightarrow a_n = 2(3)^3 + (-5)^3 = -71$ ✓

✓

Name Mohammad Yaser Awak , ID b00090174

Quiz 8, MTH 213 , Fall 2022

Ayman Badawi

15/15

QUESTION 1. The following numbers will be used to create car license plates: 2, 3, 4, 5, 6, 7, 8. Each plate number must have six digits.

2/2

a) In the event where repetition is prohibited, how many **even** plate numbers can be created?

$2\times3\times4\times5\times6\times1 + 2\times3\times4\times5\times6\times1 + 2\times3\times4\times5\times6\times1 + 2\times3\times4\times5\times6\times1$

$= 2880$ ✓

2/2

b) How many possible license plate numbers may be made if adjacent digits must differ?

$7\times6\times6\times6\times6\times6 = 54432$ ✓

2/2

c) There are 3 men and 4 women in the class, so we need to form a committee of 4 persons. How many different ways can we set up this committee so that there is at least one woman on it? [Hint: stare carefully]

3M and 1W or 2M and 2W or 1M and 3W or 4W

$3C3 \cdot 4C1 + 3C2 \cdot 4C2 + 3C1 \cdot 4C3 + 4C4 = 35$ ✓

QUESTION 2. Let $a_n = 3a_{n-1} + 4a_{n-2} + 6n + 2$. Find a general formula for a_n[no need to find c_1, c_2]

$cn+d$

$a_h(n)$:

$a_n - 3a_{n-1} - 4a_{n-2} = 0$

$$\frac{\alpha^n - 3\alpha^{n-1} - 4\alpha^{n-2}}{\alpha^{n-2}} = \frac{0}{\alpha^{n-2}}$$

$\alpha^2 - 3\alpha - 4 = 0$

$\alpha = 4 \ , \ \alpha = -1$

$a_h(n) = c_1(4)^n + c_2(-1)^n$ ✓

4/4

$a_p(n)$:

$a_p(n) - 3a_p(n-1) - 4a_p(n-2) = 6n+2$

$Cn+d - 3(C(n-1)+d) - 4(c(n-2)+d) = 6n+2$

$cn+d - 3(Cn - C + d) - 4(cn - 2C + d) = 6n+2$

$\underline{cn} \underline{\underline{+d}} \underline{-3Cn} + \underline{\underline{\underline{3C}}} \underline{\underline{-3d}} \underline{-4Cn} + \underline{\underline{\underline{8C}}} \underline{\underline{-4d}} = 6n+2$

$-6Cn \boxed{-6d+11C} = 6n \boxed{+2}$

$-6Cn = 6n \Rightarrow C = -1$

$-6d + 11C = 2 \Rightarrow d = \frac{13}{-6}$ ✓

$\Rightarrow a_p(n) = -n - \frac{13}{6}$ ✓

$$\boxed{a_n = c_1(4)^n + c_2(-1)^n - n - \frac{13}{6}}$$

1/1

4/4

Faculty information

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15/15

Quiz 9, MTH 213 , Fall 2022

Ayman Badawi

QUESTION 1.

5/5

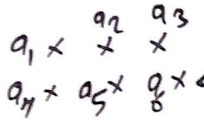
Can we construct a graph of order 8 with the following sequence of degrees: 4, 4, 1,1,1, 1, 1, 1? explain. THEN DRAW SUCH GRAPH

Hakimi algorithm : 4, 4, 1, 1, 1, 1, 1, 1

3, 0, 0, 0, 1, 1, 1

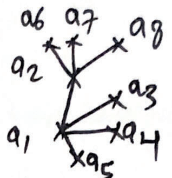
3, 1, 1, 1, 0, 0, 0

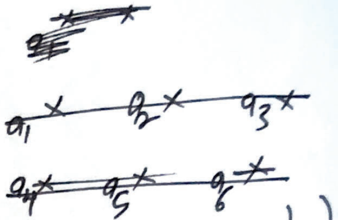
0, 0, 0, 0, 0, 0



we can

✓





(original graph)

Yes, we can. according to the algorithm! ✓

QUESTION 2.

Can we construct a graph of order 6 with the following sequence of degrees: 4, 4, 4, 3, 3, 2? Explain.

5/5

4, 4, 4, 3, 3, 2

3, 3, 2, 2, 2

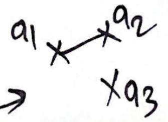
2, 1, 1, 2

2, 2, 1, 1

1, 0, 1

1, 1, 0

we can



✓

Yes we can according to the Hakimi algorithm.

QUESTION 3.

5/5

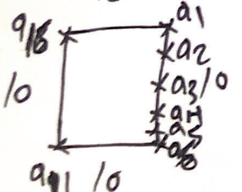
A crowd of 1203 persons. The only thing we know that all of them were born in 2000. You said " there are at least m persons were born on the same day and the same month. Consider that each month has 30 days. What is the best value of m so your statement is correct?

2/2

$$m = \left\lceil \frac{1203}{30 \times 12} \right\rceil = 4$$ ✓

(b) A square has length 10. You need to plot points (randomly) on the sides of the square so that there are at least 2 points, say A, B, where the distance between A, B is strictly less than 2. What is the minimum number of points that you need to plot?

3/3



$$\text{minimum no. of pts.} = \frac{4 \times 10}{2} + 1 = 21$$ ✓

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